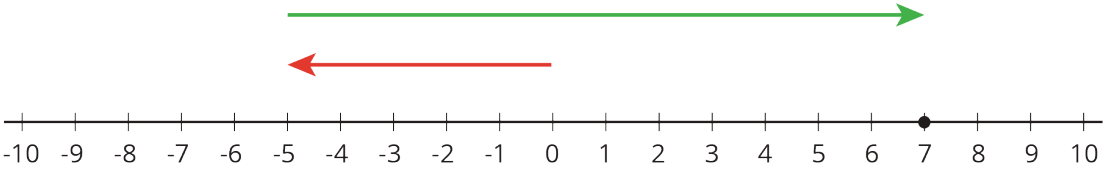
Adding and Subtracting Rational Numbers

Family Support Materials 1

This week your student will be adding and subtracting with negative numbers. We can represent this on a number line using arrows. The arrow for a positive number points right, and the arrow for a negative number points left. We add numbers by putting the arrows tail to tip.

For example, here is a number line that shows $-5 + 12 = 7$.



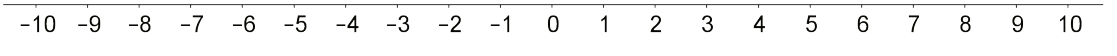
The first number is represented by an arrow that starts at 0 and points 5 units to the left. The next number is represented by an arrow that starts directly above the tip of the first arrow and points 12 units to the right. The answer is 7 because the tip of this arrow ends above the 7 on the number line.

In elementary school, students learned that every addition equation has two related subtraction equations. For example, if we know $3 + 5 = 8$, then we also know $8 - 5 = 3$ and $8 - 3 = 5$.

The same thing works when there are negative numbers in the equation. From the previous example, $-5 + 12 = 7$, we also know $7 - 12 = -5$ and $7 - -5 = 12$.

Here is a task to try with your student:

1. Use the number line to show $3 + -5$.

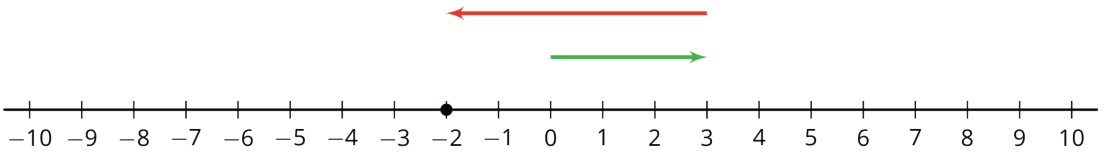


2. What does your answer tell you about the value of:
   a. $-2 - 3$?

   b. $-2 - -5$?

Solution:

1. The first arrow starts at 0 and points 3 units to the right. The next arrow starts at the tip of the first arrow and points 5 units to the left. This arrow ends above the -2, so $3 + -5 = -2$.



2. From the addition equation $3 + -5 = -2$, we get the related subtraction equations:

   a. $-2 - 3 = -5$

   b. $-2 - -5 = 3$

Multiplying and Dividing Rational Numbers

Family Support Materials 2

This week your student will be multiplying and dividing with negative numbers. The rules for multiplying positive and negative numbers are designed to make sure that addition and multiplication work the same way they always have.

For example, in elementary school students learned to think of "4 times 3" as 4 groups of 3, like $4 \cdot 3 = 3 + 3 + 3 = 12$. We can think of "4 times -3" the same way: $4 \cdot \text{-}3 = (\text{-}3) + (\text{-}3) + (\text{-}3) + (\text{-}3) = \text{-}12$. Also, an important property of multiplication is that we can multiply numbers in either order. This means that $\text{-}3 \cdot 4 = 4 \cdot \text{-}3 = \text{-}12$.

What about $\text{-}3 \cdot \text{-}4$? It may seem strange, but the answer is 12. To understand why this is, we can think of -4 as $(0 - 4)$.

$$(\text{-}3) \cdot (\text{-}4)$$

$$(\text{-}3) \cdot (0 - 4)$$

$$(\text{-}3 \cdot 0) - (\text{-}3 \cdot 4)$$

$$0 - \text{-}12$$

$$12$$

After more practice, your student will be able to remember this without needing to think through examples:

- A positive times a negative is a negative.
- A negative times a positive is a negative.
- A negative times a negative is a positive.

Here is a task to try with your student:

1. Calculate $5 \cdot \text{-}2$.
2. Use your answer to the previous question to calculate:
   a. $\text{-}2 \cdot 5$
   b. $\text{-}2 \cdot \text{-}5$
   c. $\text{-}5 \cdot \text{-}2$

Solution:

1. The answer is -10. We can think of $5 \cdot -2$ as 5 groups of -2, so
   $5 \cdot -2 = (-2) + (-2) + (-2) + (-2) + (-2) = -10$

2.
   a. The answer is -10. We can multiply numbers in either order, so
      $-2 \cdot 5 = 5 \cdot -2 = -10$

   b. The answer is 10. We can think of -5 as $(0 - 5)$, and $-2 \cdot (0 - 5) = 0 - -10 = 10$.

   c. The answer is 10. Possible Strategies:
      - We can think of -2 as $(0 - 2)$, and $-5 \cdot (0 - 2) = 0 - -10 = 10$.
      - We can multiply numbers in either order, so $-5 \cdot -2 = -2 \cdot -5 = 10$.

Four Operations with Rational Numbers

Family Support Materials 3

This week your student will use what they know about negative numbers to solve equations.

- The *opposite* of 5 is -5, because $5 + \text{-}5 = 0$. This is also called the additive inverse.
- The *reciprocal* of 5 is $\frac{1}{5}$, because $5 \cdot \frac{1}{5} = 1$. This is also called the multiplicative inverse.

Thinking about opposites and reciprocals can help us solve equations. For example, what value of x makes the equation $x + 11 = \text{-}4$ true?

$$\begin{aligned} x + 11 &= \text{-}4 \\ x + 11 + \text{-}11 &= \text{-}4 + \text{-}11 \\ x &= \text{-}15 \end{aligned}$$

11 and -11 are opposites.

The solution is -15.

What value of y makes the equation $\frac{\text{-}1}{3}y = 6$ true?

$$\begin{aligned} \frac{\text{-}1}{3}y &= 6 \\ \text{-}3 \cdot \frac{\text{-}1}{3}y &= \text{-}3 \cdot 6 \\ y &= \text{-}18 \end{aligned}$$

$\frac{\text{-}1}{3}$ and -3 are reciprocals.

The solution is -18.

Here is a task to try with your student:

Solve each equation:

$25 + a = 17$ $\quad$ $\text{-}4b = \text{-}30$ $\quad$ $\frac{\text{-}3}{4}c = 12$

Solution:

1. -8, because $17 + \text{-}25 = \text{-}8$.
2. 7.5 or equivalent, because $\frac{\text{-}1}{4} \cdot \text{-}30 = 7.5$.
3. -16, because $\frac{\text{-}4}{3} \cdot 12 = \text{-}16$.